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An analytical study of laminar counterflow double-pass heat exchangers with external refluxes

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Abstract

A new device of inserting an insulation sheet or an impermeable sheet with negligible thermal resistance to divide an open duct into two channels with uniform wall temperature and with external refluxes at the ends, resulting in substantially improving the heat transfer, has been studied and investigated by an orthogonal expansion technique. The analytical results are represented graphically and compared with that in an open conduit of the same size without recycle. Considerable improvement in heat transfer is obtainable by employing double-pass operations with an impermeable sheet of negligible thermal resistance instead of using double-pass operations with an insulation sheet inserted and single-pass operations. The effect of sheet location on the enhancement of heat transfer efficiency as well as on the increment of power consumption, has been also discussed. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Conjugated Graetz problems; Orthogonal expansion techniques; Heat transfer; External refluxes

1. Introduction

Analysis of laminar forced convection heat transfer in a circular tube with neglecting the effect of axial conduction is known as the Graetz problem [1,2]. Applications of multistream or multiphase problems such as heat exchangers with external refluxes have led to improved performance in convection heat transfer in multipass operations [3–5], as in conjugated Graetz problems, are coupled through mutual conditions at the boundaries. Recently, numerous notable papers have been studied to attack the difficulties associated with analytical solution of the equation for heat transfer in these problems. Perelman [6] pointed out a general theory of equations on conjugated problems of the hyperbolic-elliptic type. Murkerjee and Davis [7] proposed a simple method to analyze the temperature distribution of a stratified two-phase laminar flow. Kim and Coony [8] solved the conjugated boundary value problem involving chemical reaction in hollow-fiber enzyme reactor. Another conjugated boundary value problems in chemical engineering application were treated by Davis and Venkatesh [9]. Among problems involving any dual combination of fluid and solid phases, Papoutsakis and Ramkrishna [10,11] have developed and presented a general formalism. Studies on the simultaneous energy equations in the fluid and the solid regions by utilizing eigenfunction expansions have been solved by Yin and Bau [12].

The availability of the recycle-effect concept in multipass operations heat transfer through a parallel-plate device is technically and economically feas-

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Nomenclature

B conduit width, m
F_m eigenfunction associated with eigenvalue λ_m
f friction factor
G_Z Graetz number, $VW/\alpha BL$
G_m function defined during the use of orthogonal expansion method
$\overline{h}$ average heat transfer coefficient, kW/m^2 K
I_h improvement of heat transfer, defined by Eqs. (33) and (34)
I_p increment of power consumption, defined by Eq. (35)
k thermal conductivity of the fluid kW/m K
L conduit length, m
lw_f friction loss in conduit, kJ/kg
$\overline{Nu}$ Nusselt number
R reflux ratio, reverse volume flow rate divided by input volume flow rate
Re Reynolds number
S_m expansion coefficient associated with eigenvalue λ_m
T temperature of fluid, K
V input volume flow rate of conduit, m^3/s
v velocity distribution of fluid, m/s
$\overline{v}$ average velocity of fluid, m/s
W distance between two parallel plates, m
x transversal coordinate, m
z longitudinal coordinate, m

Greek symbols
α thermal diffusivity of fluid, m^2/s
β ratio of channel thickness, W_a/W
η transversal coordinate, x/W
θ dimensionless temperature, $(T-T_i)/(T_s-T_i)$
λ_m eigenvalue
ξ longitudinal coordinate, z/L
ρ density of the fluid, kg/m^3
ψ dimensionless temperature, $(T-T_s)/(T_i-T_s)$

Superscripts and subscripts
a in forward flow channel
b in backward flow channel
F at the outlet of a double-pass device
i at the inlet
L at the outlet, $\xi = 1$
0 in a single-pass device without recycle
s at the wall surface
t in a double-pass device with inserting an insulation sheet

ible. The theoretical formulations of Graetz problems and conjugated Graetz problems by use of orthogonal expansion techniques [13–20] involved an infinite number of eigenvalues and only the first negative eigenvalue was considered for the rapid convergence in the present paper. It was found that the application of inserting in parallel an impermeable sheet to divide an open duct into two channels for double-pass operation creates the driving force for heat transfer from upper channel to lower channel, as shown in Fig. 1. Therefore, the fluid in lower channel is heated on both sides, and hence the extent of further improvement in transfer efficiency will increase the outlet fluid in upper channel, leading to improved performance.

The purpose of the present study is to investigate the improvement of performance in double-pass parallel-plate heat exchangers by inserting an impermeable sheet with negligible thermal resistance or an insulation sheet. The solutions to these problems are obtained by using the method of separation of variables, where the resulting eigenvalue problem is solved by the orthogonality conditions.

2. The governing equations for temperature distributions

An impermeable sheet with negligible thermal resistance or an insulation sheet is inserted in parallel into a parallel-plate channel with thickness W, length L, and width B ($\gg W$) to divide the open duct into two parts, subchannel a (lower channel) and subchannel b (upper channel), with thickness βW and $(1-\beta)W$, respectively, as shown in Fig. 1. Before entering subchannel a, the fluid with volume flow rate V and inlet temperature T_i will premix the fluid flowing out from the end of subchannel a with the volume flow rate RV and outlet temperature T_L, which is then conducted by a pump situated at the end of subchannel a.

The following assumptions are made in present analysis: constant physical properties and wall temperatures, purely fully-developed laminar flow in each subchannel, and negligible end effects and axial diffusion. After the following dimensionless variables are introduced:

$$\eta_a = \frac{x_a}{W_a}, \quad \eta_b = \frac{x_b}{W_b}, \quad \xi = \frac{z}{L},$$

$$G_Z = \frac{V(W_a + W_b)}{\alpha BL} = \frac{VW}{\alpha BL}, \quad W_a = \beta W, \tag{1}$$

$$W_b = (1-\beta)W$$

The velocity distributions in dimensionless form may be written as

$$v_a(\eta_a) = \overline{v_a}\left(6\eta_a - 6\eta_a^2\right), \quad 0 \le \eta_a \le 1 \tag{2}$$

$$v_b(\eta_b) = \overline{v_b}\left(6\eta_b - 6\eta_b^2\right), \quad 0 \le \eta_b \le 1 \tag{3}$$

in which

$$\overline{v_a} = \left[\frac{V(R+1)}{W_a B}\right], \qquad \overline{v_b} = -\left[\frac{V}{W_b B}\right].$$

2.1. An insulation sheet inserted

For the double-pass device, an insulation sheet is inserted, as shown in Fig. 1. The equations of energy in dimensionless form may be obtained as

$$\frac{\partial^2 \psi_a(\eta_a, \xi)}{\partial \eta_a^2} = \left(\frac{W_a^2 v_a}{L\alpha}\right) \frac{\partial \psi_a(\eta_a, \xi)}{\partial \xi} \tag{4}$$

$$\frac{\partial^2 \psi_b(\eta_b, \xi)}{\partial \eta_b^2} = \left(\frac{W_b^2 v_b}{L\alpha}\right) \frac{\partial \psi_b(\eta_b, \xi)}{\partial \xi} \tag{5}$$

in which

$$\psi_a = \frac{T_a - T_s}{T_i - T_s}, \quad \psi_b = \frac{T_b - T_s}{T_i - T_s}, \quad \theta_a = 1 - \psi_a, \quad \theta_b = 1 - \psi_b \tag{6}$$

The boundary conditions for solving Eqs. (4) and (5) are

$$\psi_a(0, \xi) = 0 \tag{7}$$

$$\psi_b(0, \xi) = 0 \tag{8}$$

$$\left.\frac{\partial \psi_a}{\partial \eta_a}\right|_{\eta_a=1} = 0 \tag{9}$$

$$\left.\frac{\partial \psi_b}{\partial \eta_b}\right|_{\eta_b=1} = 0 \tag{10}$$

By following the similar mathematical treatment performed in the our previous works [4,5], the analytical solution to this type of problem will be obtained by use of an orthogonal expansion technique. The dimensionless outlet temperature which are referred to as the

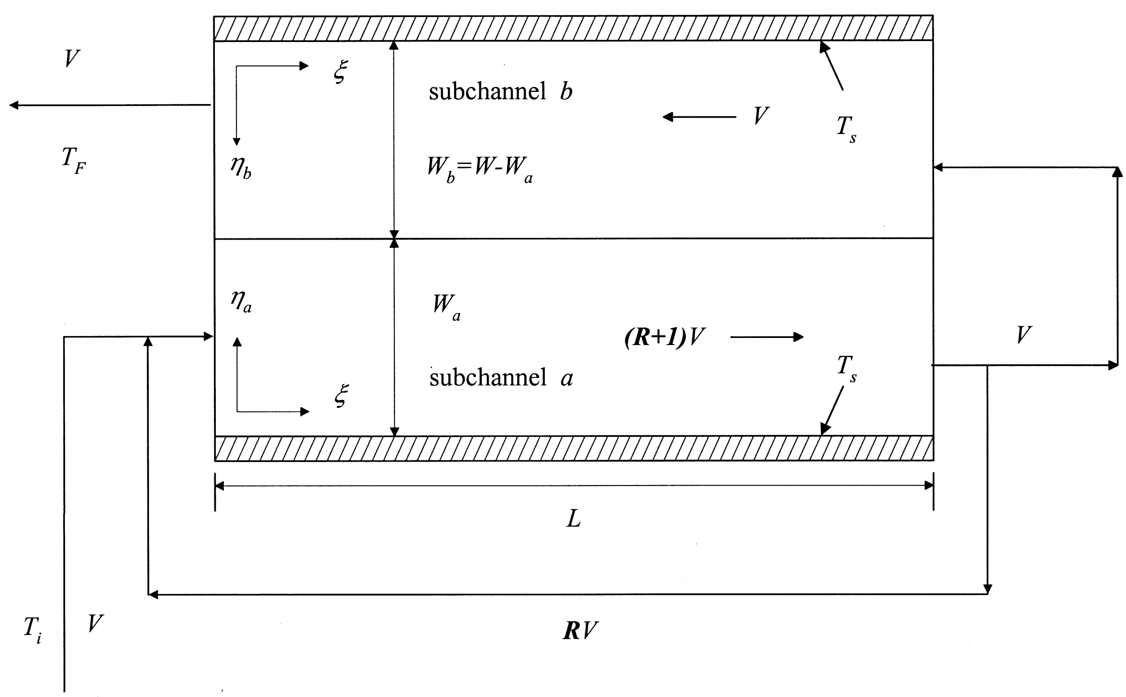

Fig. 1. Double-pass parallel-plate heat exchangers with external refluxes at both ends.

bulk temperature, are obtained by making the overall energy balances on both subchannels. It was in terms of he Graetz number (G_Z), eigenvalues ($\lambda_{a,m}$ and $\lambda_{b,m}$), expansion coefficients ($S_{a,m}$ and $S_{b,m}$), location of impermeable sheet (β) and eigenfunctions $F_{a,m}(\eta_a)$ and $F_{b,m}(\eta_b)$). The result is as follows:

$$V(1-\psi_F) = \int_0^1 \frac{\alpha BL}{W_a}\frac{\partial \psi_a(0,\xi)}{\partial \eta_a}\,d\xi + \int_0^1 \frac{\alpha BL}{W_b}\frac{\partial \psi_b(0,\xi)}{\partial \eta_b}\,d\xi = \left[\frac{\alpha BL}{W_a}\sum_{m=0}^{\infty} S_{a,m}F'_{a,m}(0)\int_0^1 e^{-\lambda_{a,m}(1-\xi)}\,d\xi + \frac{\alpha BL}{W_b}\sum_{m=0}^{\infty} S_{b,m}F'_{b,m}(0)\int_0^1 e^{-\lambda_{b,m}(1-\xi)}\,d\xi\right] \quad (11)$$

then

$$\psi_F = 1 - \frac{1}{G_Z}\left[\sum_{m=0}^{\infty}\frac{(1-e^{-\lambda_{a,m}})}{\lambda_{a,m}\beta}S_{a,m}F'_{a,m}(0) + \sum_{m=0}^{\infty}\frac{(1-e^{-\lambda_{b,m}})}{\lambda_{b,m}(1-\beta)}S_{b,m}F'_{b,m}(0)\right] \quad (12)$$

After the eigenvalues and associated expansion coefficients are obtained, the dimensionless outlet temperatures (ψ_F and $\theta_F = 1-\psi_F$) are readily calculated by Eq. (12). After the coefficients, $S_{a,m}$ and $S_{b,m}$ are obtained. Also, the mixed inlet temperature of the lower channel is calculated by

$$\psi_a(\eta_a,0) = \frac{\left(\frac{R}{R+1}\right)\int_0^1 v_a W_a B\psi_a(\eta_a,1)\,d\eta_a + V}{V(R+1)} = \frac{1}{R+1}\left[1+\left(\frac{R}{R+1}\right)\frac{W_a B}{V}\times\sum_{m=0}^{\infty}S_{a,m}\int_0^1 v_a(\eta_a)F_{b,m}(\eta_a)\,d\eta_a\right] = \frac{1}{R+1}\left[1+\frac{1}{G_Z\beta}\left(\frac{R}{R+1}\right)\sum_{m=0}^{\infty}\left(\frac{S_{a,m}}{\lambda_{a,m}}\right)\times\left\{F'_{a,m}(1)-F'_{a,m}(0)\right\}\right] \quad (13)$$

2.2. An impermeable sheet inserted with negligible thermal resistance

Similarly, for the double-pass device, an impermeable sheet with negligible thermal resistance inserted as shown in Fig. 1. The equations of energy in dimensionless form may also be obtained as

$$\frac{\partial^2 \psi_{a,t}(\eta_a,\xi)}{\partial \eta_a^2} = \left(\frac{W_a^2 v_a}{L\alpha}\right)\frac{\partial \psi_{a,t}(\eta_a,\xi)}{\partial \xi} \quad (14)$$

$$\frac{\partial^2 \psi_{b,t}(\eta_b,\xi)}{\partial \eta_b^2} = \left(\frac{W_b^2 v_b}{L\alpha}\right)\frac{\partial \psi_{b,t}(\eta_b,\xi)}{\partial \xi} \quad (15)$$

in which

$$\psi_{a,t} = \frac{T_{a,t}-T_s}{T_i-T_s},\quad \psi_{b,t} = \frac{T_{b,t}-T_s}{T_i-T_s},\quad \theta_{a,t} = 1-\psi_{a,t},\quad \theta_{b,t} = 1-\psi_{b,t} \quad (16)$$

The boundary conditions for solving Eqs. (14) and (15) are

$$\psi_{a,t}(0,\xi) = 0 \quad (17)$$

$$\psi_{b,t}(0,\xi) = 0 \quad (18)$$

$$\psi_{a,t}(1,\xi) = \psi_{b,t}(1,\xi) \quad (19)$$

$$-\frac{\partial \psi_{a,t}(1,\xi)}{\partial \eta_a} = \frac{W_a}{W_b}\frac{\partial \psi_{b,t}(1,\xi)}{\partial \eta_b} \quad (20)$$

The mathematical treatment is similar to that in the previous section, except the orthogonality conditions of conjugated Graetz problems. the dimensionless inlet and outlet temperatures for double-pass devices by inserting an impermeable sheet with negligible thermal resistance ($\theta(\eta_a,0)$ and $\theta_{F,t}$) were also obtained in terms of the Graetz number (G_Z), eigenvalues ($\lambda_{m,t}$), expansion coefficients ($S^t_{a,m}$ and $S^t_{b,m}$), location of impermeable sheet (β) and eigenfunctions ($F^t_{a,m}(\eta_a)$ and $F^t_{b,m}(\eta_b)$). The results are

$$\theta_{F,t} = 1-\psi_{F,t} = \frac{1}{G_Z}\left[\sum_{m=0}^{\infty}\frac{(1-e^{-\lambda_{m,t}})}{\lambda_{m,t}\beta}S^t_{a,m}F^{t\prime}_{a,m}(0) + \sum_{m=0}^{\infty}\frac{(1-e^{-\lambda_{m,t}})}{\lambda_{m,t}(1-\beta)}S^t_{b,m}F^{t\prime}_{b,m}(0)\right] \quad (21)$$

and

$$\psi_{a,t}(\eta_a, 0) = \frac{\left(\frac{R}{R+1}\right)\int_0^1 v_a W_a B \psi_{a,t}(\eta_a, 1)\, d\eta_a + V}{V(R+1)}$$
$$= \frac{1}{R+1}\left[1 + \left(\frac{R}{R+1}\right)\frac{W_a B}{V}\sum_{m=0}^{\infty} S^t_{a,m} \times \int_0^1 v_a(\eta_a) F^t_{a,m}(\eta_a)\, d\eta_a\right]$$
$$= \frac{1}{R+1}\left[1 + \frac{1}{G_Z \beta}\left(\frac{R}{R+1}\right) \times \sum_{m=0}^{\infty}\left(\frac{S^t_{a,m}}{\lambda_{m,t}}\right)\left\{F^{t\prime}_{a,m}(1) - F^{t\prime}_{a,m}(0)\right\}\right] \tag{22}$$

2.3. Temperature distribution in a single-pass device without recycle

For the single-pass device of same size without recycle, as shown in Fig. 2. The velocity distribution and equation of energy in dimensionless form may then be written as

$$\frac{\partial^2 \psi_0(\eta_0, \xi)}{\partial \eta_0^2} = \left(\frac{W^2 v_0(\eta_0)}{L\alpha}\right)\frac{\partial \psi_0(\eta_0, \xi)}{\partial \xi} \tag{23}$$

$$v_0(\eta_0) = \frac{V}{WB}(6\eta_0 - 6\eta_0^2), \quad 0 \le \eta_0 \le 1 \tag{24}$$

in which

$$\eta_0 = \frac{x}{W}, \quad \xi = \frac{z}{L}, \quad \psi_0 = \frac{T_0 - T_s}{T_i - T_s}, \quad \theta_0 = 1 - \psi_0 \tag{25}$$

The boundary conditions for solving Eq. (23) are

$$\psi_0(0, \xi) = 0 \tag{26}$$

$$\psi_0(1, \xi) = 0 \tag{27}$$

Also, the dimensionless inlet temperature would be the constrain for whole system.

$$\psi_0(\eta_0, 0) = 1 = \frac{\int_0^1 v_0 WB \psi_0(\eta_0, 0)\, d\eta_0}{V}$$
$$= \frac{WB}{V}\sum_{m=0}^{\infty} e^{-\lambda_{0,m}} S_{0,m} \int_0^1 v_0(\eta_0) F_{0,m}(\eta_0)\, d\eta_0$$
$$= \frac{1}{G_Z}\sum_{m=0}^{\infty}\left(\frac{e^{-\lambda_{0,m}} S_{0,m}}{\lambda_{0,m}}\right)\left\{F'_{0,m}(1) - F'_{0,m}(0)\right\} \tag{28}$$

The calculation procedure for a single-flow device is rather simpler than that for a double-pass device. The outlet temperature for single-flow devices ($\theta_{0,F}$) was also obtained in terms of the Graetz number (G_Z),

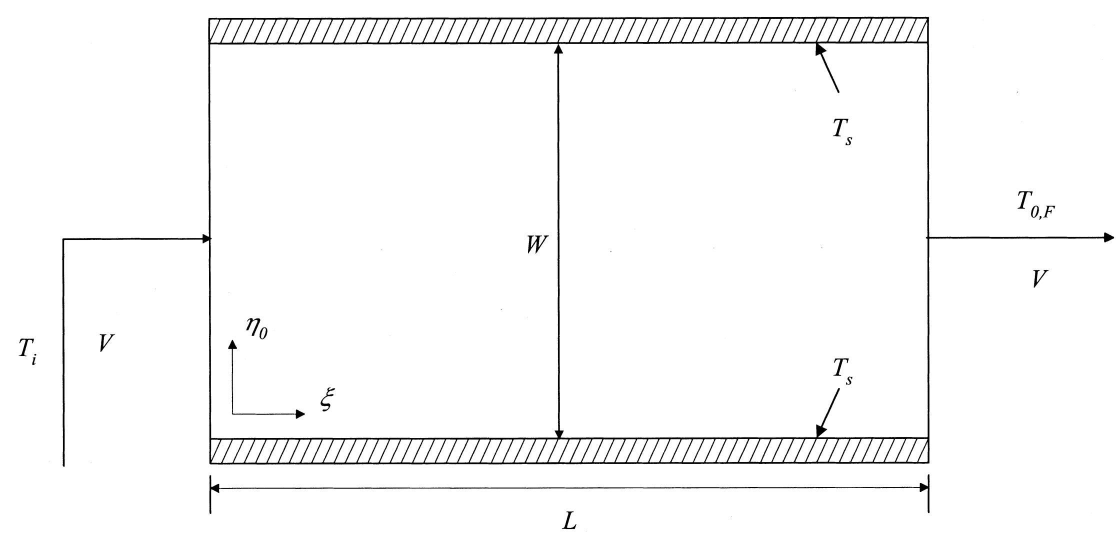

Fig. 2. Single-pass heat exchangers without recycle.

Table 1
The improvement of the transfer efficiency with reflux ratio and barrier position as parameters of double-pass operations by inserting an insulation sheet

I_h (%)	$R = 1.0$			$R = 2.0$			$R = 5.0$		
	$\beta = 0.25$	$\beta = 0.5$	$\beta = 0.75$	$\beta = 0.25$	$\beta = 0.5$	$\beta = 0.75$	$\beta = 0.25$	$\beta = 0.5$	$\beta = 0.75$
$G_Z = 1$	−0.10	−0.04	0.05	0	−0.01	0.05	0.04	0.02	0.05
10	23.92	13.14	36.94	26.83	14.22	37.31	29.80	15.34	37.70
100	67.37	29.84	71.47	68.28	30.08	71.58	69.20	30.33	71.68
1000	76.03	32.50	76.52	76.13	32.52	76.51	76.23	32.53	76.52

eigenvalues ($\lambda_{0,m}$), expansion coefficients ($S_{0,m}$) and eigenfunction ($F_{0,m}(\eta_0)$). The result is

$$\theta_{0,F} = 1 - \psi_{0,F}$$
$$= \frac{1}{G_Z}\sum_{m=0}^{\infty}\left[\frac{(1-e^{-\lambda_{0,m}})}{\lambda_{0,m}} S_{0,m} F'_{0,m}(0) - \frac{(1-e^{-\lambda_{0,m}})}{\lambda_{0,m}} S_{0,m} F'_{0,m}(1)\right] \quad (29)$$

3. Improvement of transfer efficiency

By following the same definition in the previous works [4,5], the Nusselt number for double-pass devices by inserting an insulation sheet and by inserting an impermeable with negligible thermal resistance may be obtained as follows:

$$\overline{Nu} = \frac{\bar{h}W}{k} = \frac{VW}{2\alpha BL}\left(1-\psi_F\right) = 0.5G_Z\theta_F \quad (30)$$

for inserting an insulation sheet, and

$$\overline{Nu_t} = \frac{\bar{h}W}{k} = \frac{VW}{2\alpha BL}\left(1-\psi_{F,t}\right) = 0.5G_Z\theta_{F,t} \quad (31)$$

for an impermeable sheet with negligible thermal resistance. Similarly, for a single-pass device without recycle

$$\overline{Nu_0} = \frac{\overline{h_0}W}{k} = \frac{VW}{2\alpha BL}\left(1-\psi_{0,F}\right) = 0.5G_Z\theta_{0,F} \quad (32)$$

The improvement of heat transfer, I_h and $I_{h,t}$, for double-pass devices by inserting an insulation sheet and by inserting an impermeable sheet with negligible thermal resistance are best illustrated by calculating the percentage increase in heat-transfer rate, based on the heat transfer of a single-pass device of same Graetz number without recycle as

$$I_h = \frac{\overline{Nu} - \overline{Nu_0}}{\overline{Nu_0}} = \frac{\psi_{0,F} - \psi_F}{1-\psi_{0,F}} = \frac{\theta_F - \theta_{0,F}}{\theta_{0,F}} \quad (33)$$

and

$$I_{h,t} = \frac{\overline{Nu_t} - \overline{Nu_0}}{\overline{Nu_0}} = \frac{\psi_{0,F} - \psi_{F,t}}{1-\psi_{0,F}} = \frac{\theta_{F,t} - \theta_{0,F}}{\theta_{0,F}} \quad (34)$$

The results are shown in Tables 1 and 2.

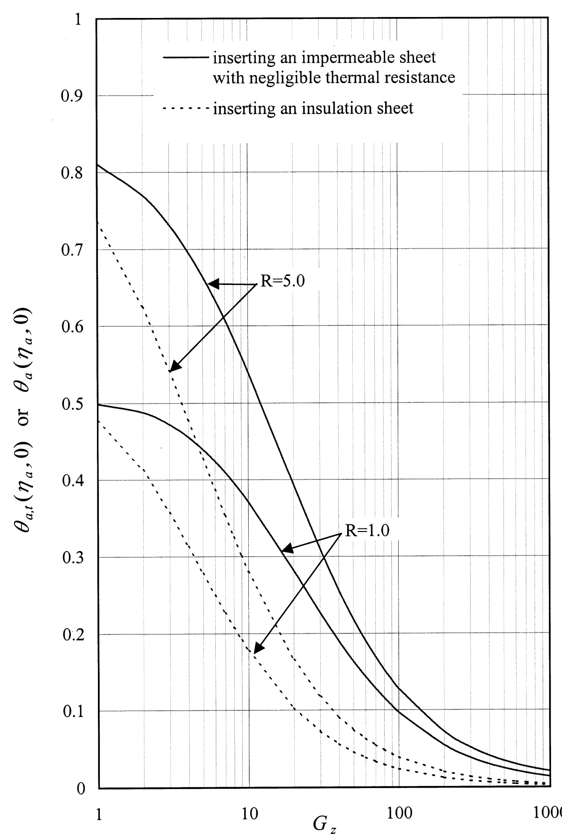

Fig. 3. Dimensionless average inlet temperature of fluid after mixing. Reflux ratio as a parameter; $\beta = 0.5$, $0 \le \eta_a \le 1$.

Table 2
The improvement of the transfer efficiency with reflux ratio and barrier position as parameters double-pass operations by inserting an impermeable sheet with negligible thermal resistance

I_h, t (%)	$R = 1.0$			$R = 2.0$			$R = 5.0$		
	$\beta = 0.25$	$\beta = 0.5$	$\beta = 0.75$	$\beta = 0.25$	$\beta = 0.5$	$\beta = 0.75$	$\beta = 0.25$	$\beta = 0.5$	$\beta = 0.75$
$G_Z = 1$	−9.51	−10.18	−7.21	−8.35	−11.18	−9.82	−13.86	−18.66	−17.51
10	28.31	15.56	31.13	31.34	17.06	32.57	33.05	18.43	34.96
100	158.76	97.93	166.31	181.27	115.62	205.70	192.64	134.57	246.27
1000	202.92	120.71	212.53	236.04	145.40	275.17	294.54	172.80	343.82

4. Results and discussion

4.1. Outlet temperature and transfer efficiency in double-pass devices

Figs. 3 and 4 show, respectively, the dimensionless inlet temperatures $\theta_a(\eta_a, 0)$ and $\theta_{a,t}(\eta_a, 0)$ of the fluid after mixing and dimensionless outlet temperatures θ_F and $\theta_{F,t}$ with the reflux ratio as a parameter for $\beta = 0.5$. It is shown in Fig. 3 that the dimensionless inlet temperature of fluid after mixing increases with reflux ratio but decreases with Graetz number. Although the

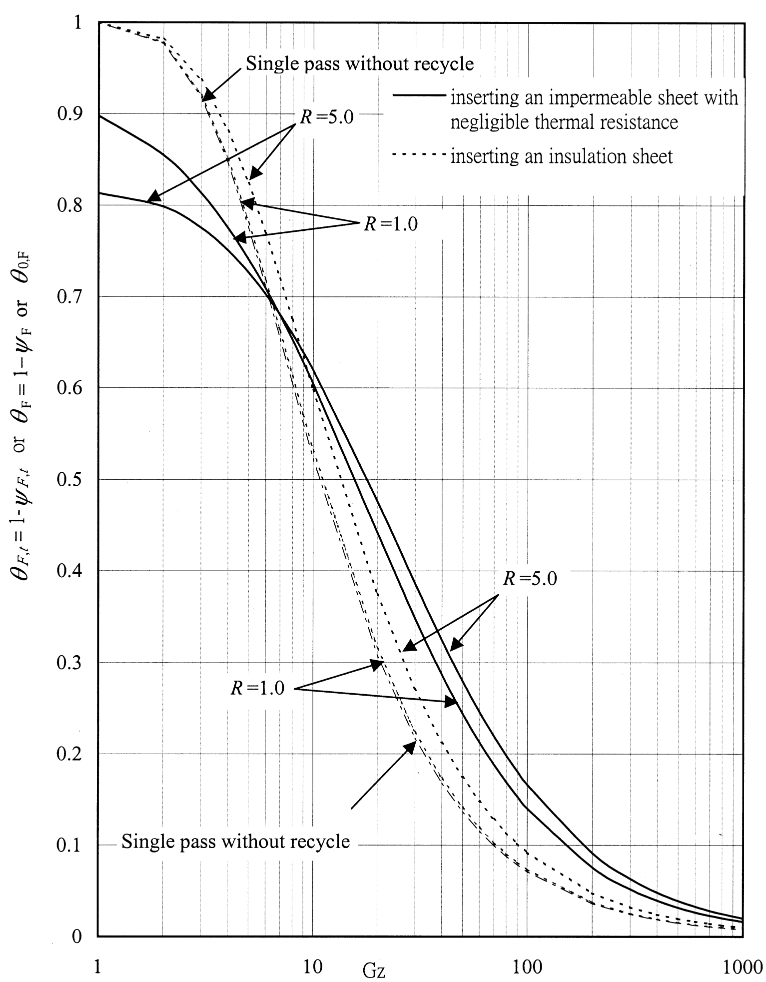

Fig. 4. Dimensionless average outlet temperature of three devices vs. G_Z with reflux ratio as a parameter; $\beta = 0.5$.

Table 3
The increment of power consumption with reflux ratio and barrier position as parameters

R	I_p		
	$\beta = 0.25$	$\beta = 0.5$	$\beta = 0.75$
0.5	97.37	19.00	66.56
1.0	129.37	23.00	67.74
2.0	193.37	31.00	70.11
5.0	385.37	55.00	77.22

recycle-effect has positive influences on the heat transfer, the preheating effect by increasing the reflux ratio cannot compensate for the decrease of residence time at low Graetz number, and hence the outlet temperature (or heat transfer) decreases with increasing reflux ratio for the device with negligible thermal resistance but not for the one with inserting an insulation sheet. It was also found in Fig. 4 and Tables 1 and 2 that for a fixed reflux ratio, the dimensionless average outlet temperature of double-pass operations decreases also with increasing the Graetz number owing to the short residence time of fluid. For small Graetz number, say $G_Z < 10$, no improvement in transfer efficiency can be achieved in the double-pass device with negligible thermal resistance. Under this region, the double-pass device with inserting an insulation sheet is preferred to be employed rather than using the double-pass one with negligible thermal resistance operating at such conditions.

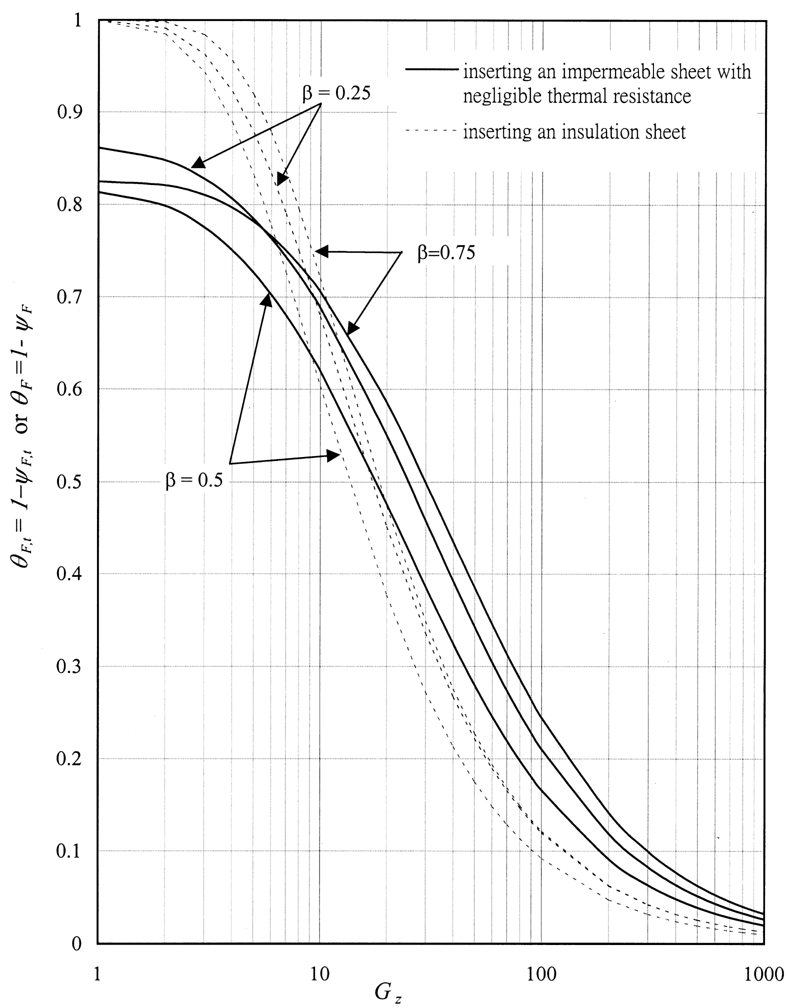

Fig. 5. Dimensionless average outlet temperature of both double-pass devices vs. G_Z with β as a parameter; $R = 5$.

Nusselt numbers and hence the improvement of transfer efficiencies are proportional to θ_F ($\theta_{F,t}$ or $\theta_{0,F}$), as shown in Eqs. (30)–(34), so the higher improvement of performance is really obtained by employing a double-pass device with negligible thermal resistance, instead of using an insulation sheet for large Graetz number. It is seen from Figs. 4 and 5 that the difference $(\theta_F - \theta_{F,t})$ of outlet temperatures decreases for small G_Z values, but then turns to minus sign with any values of R and β. Fig. 6 shows the average Nusselt numbers with the reflux ratio as a parameter for $\beta = 0.5$ while Fig. 7 and Tables 1 and 2 with the ratio of the thickness β as a parameter. It is concluded that Nusselt number increases with increasing R, but with β going away from 0.5, especially for $\beta > 0.5$.

4.2. Improvement in transfer efficiency based on a single-pass device without recycle as well as a double-pass device by inserting an insulation sheet

Figs. 4 and 5 show the dimensionless outlet temperatures, $\theta_{F,t}$, θ_F and $\theta_{0,F}$. It is found in Figs. 4 and 5 that all $\theta_{F,t}$, θ_F and $\theta_{0,F}$ decrease with increasing the Graetz number G_Z owing to the short residence time of fluid (either large V or small L) while $\theta_{F,t}$ and θ_F increase as the ratio of the thickness β goes away from 0.5, especially for $\beta > 0.5$. Figs. 6 and 7 give the graphical representation of average Nusselt number, $\overline{Nu_t}$, $\overline{Nu}$ and $\overline{Nu_0}$. It is seen in Figs. 6 and 7 that all $\overline{Nu_t}$, $\overline{Nu}$ and $\overline{Nu_0}$ increase with G_Z while they increase also when β goes away from 0.5, especially for $\beta > 0.5$. On the other hand, as shown in Fig. 6, $(\overline{Nu_t} - \overline{Nu_0})$ and $(\overline{Nu} - \overline{Nu_0})$ increase with G_Z, but the increase with G_Z is limited as G_Z approaches infinity. The improvement of heat transfer, I_h and $I_{h,t}$, in the double-pass operations are shown in Tables 1 and 2 with Graetz number and the ratio of thickness β as parameters. The minus signs in Tables 1 and 2 indicate that the single-pass device without recycle is preferred to be employed rather than using double-pass devices with negligible thermal resistance and with inserting an insulation sheet operating at such conditions. It is noted that the improvement of heat transfer, I_h and $I_{h,t}$, increase also with Graetz number as well as when β goes away from 0.5, especially for $\beta > 0.5$.

Finally, if the laminar flow in the flow channels is assumed, the increment of power consumption I_p due to the friction losses $lw_{f,a}$ and $lw_{f,b}$ for double pass while $lw_{f,0}$ for single pass) in the conduits can readily derived as [5]

$$I_p = \frac{(lw_{f,a} + lw_{f,b}) - (lw_{f,0})}{lw_{f,0}} = \frac{R+1}{\beta^3} + \frac{1}{(1-\beta)^3} - 1 \qquad (35)$$

Though the increment of power consumption does not depend on Graetz number, it increases as β goes away from 0.5. It is readily obtained from Eq. (35) that I_p increases with reflux ratio as well as with β going away from 0.5, and results for I_p are presented in Table 3.

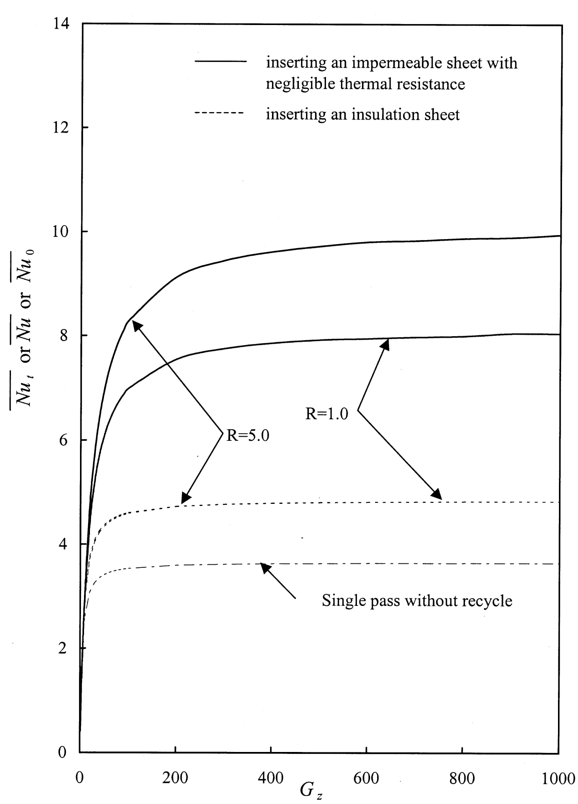

Fig. 6. Average Nusselt number of three devices vs. G_Z with reflux ratio as a parameter; $\beta = 0.5$.

5. Conclusion

Heat transfer through double-pass parallel-plate operations with an impermeable sheet of negligible thermal resistance and with an insulation sheet inserted have been investigated analytically with ignoring axial conduction or diffusion. For axial fluid conduction to be ignored, the flow Peclet number should be high enough, say $Pe > 50$. Since Graetz number is defined as $G_Z = Pe(W/L)$, for small values of G_Z, in order to neglect axial fluid conduction or diffusion, (W/L) ratio should not exceed some certain values (i.e., $G_Z = 1$, W/L should be less than 0.02). Therefore, for small G_Z number flows, for this assumption to be valid the

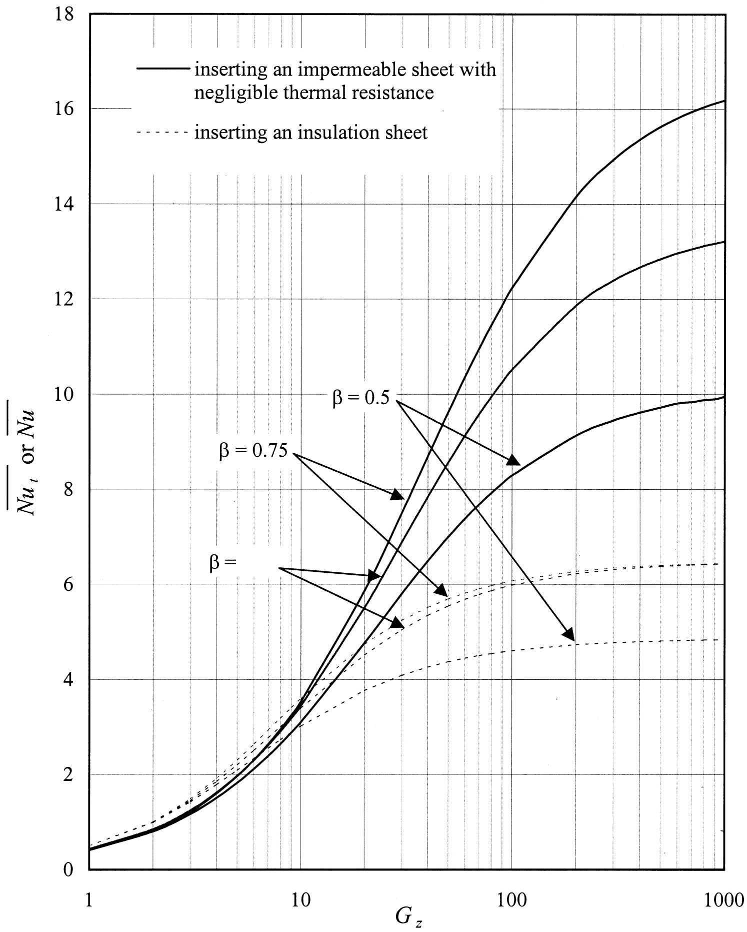

Fig. 7. Average Nusselt number both double-pass devices vs. G_Z with as a parameter; $R = 5$.

limiting values of W/L should be taken into consideration.

The Nusselt number defined in Ref. [1] is $Nu = \frac{\bar{h}D_h}{k}$, where D_h is hydraulic diameter, and the values of Nusselt number obtained in the case with constant wall temperature is 7.541. In the present study, the Nusselt number is defined in Eq. (32), $Nu = \frac{\bar{h}W}{k}$, for single-pass operation, because $D_h = 2W$ and the value of the Nusselt number with the same condition should be 3.7705. The agreement of the theoretical results in Fig. 6 with the data in Ref. [1] is also pretty good. Moreover, the Nusselt numbers, Figs. 6 and 7, of double-pass operations with an impermeable sheet of negligible thermal resistance and with an insulation inserted in the present study of recycle type are much higher than 3.7705 when $G_Z > 10$. Considerable improvement of heat transfer in double-pass heat exchangers is obtainable, this is the value of the present study in designing double-pass operations with recycle.

The methods for improving the performance in heat transfer devices are either in the double-pass operation by inserting an impermeable sheet with negligible thermal resistance or by inserting an insulation sheet. One may notice in Figs. 6 and 7 that the values of $\overline{Nu_t}$ are higher than $\overline{Nu}$ at large Graetz numbers since the application of sheet conducting double-pass devices with negligible thermal resistance create the driving force of heat transfer from subchannel b to subchannel a. Therefore, the fluid in subchannel a is heated on both sides, and hence the extent of further improvement in transfer efficiency will increase in subchannel b, leading to improved performance. The effects of β on $\overline{Nu}$ and $\overline{Nu_t}$ are also shown in Fig. 7. $\overline{Nu}$ and $\overline{Nu_t}$ increase as the value of β goes away from 0.5, especially for $\beta > 0.5$. The reason why $\beta > 0.5$ is better than $\beta < 0.5$ may be considered as that the enhancement of heat transfer in subchannel b due to decreasing the thickness W_b to increase the flow velocity can compensate for the decrease of heat transfer in subchannel a with reflux ratio R due to increasing W_a to decrease the flow velocity.

The desirable preheating effect of inlet fluid and the undesirable effect of decreasing residence time are the two conflicting effects to be confronted in the application of recycle to heat exchangers. It is noted that in Fig. 4 and Tables 1 and 2 for low Graetz number, the preheating effect produced at the inlet of subchannel a can compensate for the decrease of residence time with larger reflux ratio (say $R > 1$) for double-pass operations with inserting an insulation sheet but not for double-pass operations with inserting an impermeable sheet with negligible thermal resistance. However, the introduction of reflux still has positive effects on the heat transfer for large Graetz number and the outlet temperature as well as transfer coefficient increases with increasing reflux ratio, as also shown in Fig. 4 and Tables 1 and 2. Furthermore, as shown in Tables 1 and 2, the improvement of transfer efficiency, I_h and $I_{h,t}$, increase with increasing the Graetz number as well as reflux ratio. Since the ratio of channel thickness has much influence on the heat transfer behavior, it is presented in Tables 1 and 2 that I_h and $I_{h,t}$ increase also as β goes away from 0.5, especially for $\beta > 0.5$.

The heat-exchanger effectiveness was defined as follows [21]:

$$\text{Effectiveness} = \frac{\text{actual heat transfer}}{\text{maximum possible heat transfer}} \tag{36}$$

The actual heat transfer may be computed by calculating either the energy lost by the hot fluid or the energy gained by the cold fluid, which was defined as the ca-

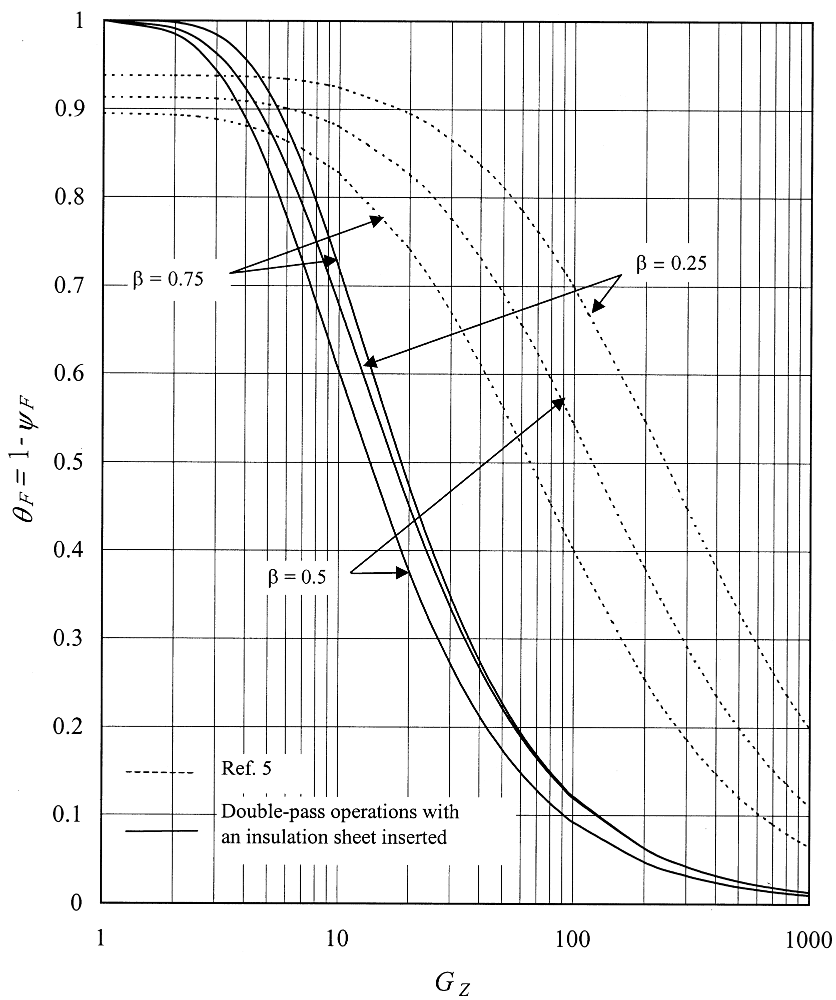

Fig. 8. Dimensionless average outlet temperature of both double-pass devices in Ref. [5] and with insulation sheet inserted in the present work vs. G_Z with as a parameter; $R = 5$.

pacity rate. The mass rate of flow was included in the capacity rate, so the exchanger effectiveness could be changed by the mass rate of flow and the mass rate of flow in the present study is a function of R and β. However, the Nusselt number in Eq. (30) (or Eq. (31)) in the present study is expressed in terms of G_Z and θ_F (or $\theta_{F,t}$), and θ_F (or $\theta_{F,t}$) represents the measurement of actual heat transfer. This is the reason why the Nusselt number changes in R, β and G_Z and could adequately reflect exchanger performance.

It is concluded that the recycle effect can enhance heat transfer for the fluid flowing through a parallel-plate channel under double-pass operations by inserting an impermeable plate with negligible thermal resistance for large Graetz number or by inserting an insulation sheet with any Graetz number. The present paper is actually the extension of other recycle problems carried out in the previous work (Ref. [5]), except the type of reflux and with two double-pass devices included. In order to explain how the present device improves on the previous work, Fig. 8 illustrates some results obtained in Ref. [5] with the same parameter values used in Fig. 5 for comparison. With this comparison, the advantage of present results is evident with small Graetz number.

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